Controlling surface morphologies by time-delayed feedback

M. Block[1],* B. Schmittmann[2], and E. Schöll[1]
1 *Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany*
2 *Department of Physics, Virginia Tech, Blacksburg, VA 24061, USA*
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We propose a new method to control the roughness of a growing surface, via a time-delayed feedback scheme. As an illustration, we apply this method to the Kardar-Parisi-Zhang equation in 1+1 dimensions and show that the effective growth exponent of the surface width can be stabilized at any desired value in the interval $[0.25, 0.33]$, for a significant length of time. The method is quite general and can be applied to a wide range of growth phenomena. A possible experimental realization is suggested.

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Introduction. The control of unstable states in chaotic or pattern-forming nonlinear dynamic systems has attracted much interest recently [1, 2]. Time-delayed feedback control [3] has been especially successful in stabilizing a variety of dynamic and spatial structures, including noise-induced oscillations and patterns found, e.g., in semiconductor nanostructures [4, 5, 6, 7]. The fabrication of such nanostructures typically involves the deposition of a material onto a substrate. One of the primary experimental goals is to achieve nanoscale control of layer thickness and surface (or interface) morphology. On the theoretical side, considerable effort has focused on developing suitable evolution equations for the growing layer and its surface [8]. While many different versions [9, 10, 11, 12, 13, 14, 15, 16] of these equations exist, depending on the details of deposition processes and molecular interactions and kinetics, all of them share certain fundamental characteristics: they are noisy, nonlinear partial differential equations in space and time, and describe an important class of generic nonequilibrium phenomena.

It is natural to ask whether the control techniques of nonlinear dynamics can be successfully applied to surface growth problems. The goal is, of course, to stabilize desired surface characteristics, such as its spatio-temporal height-height correlations or its roughness during the growth process. Even if such control can only be achieved in a finite window of time, its experimental potential is undiminished since the deposition process can simply be terminated at the desired time, thanks to today's precise in situ characterization capabilities. In this letter, we provide a first set of answers to this question. We choose the most promising type of control, time-delayed feedback, and study its effects on a paradigmatic growth model, the Kardar-Parisi-Zhang (KPZ) equation [17]. Specifically, we attempt to control the *effective* dynamic growth exponent β associated with the roughness of the growing surface. Implementing two realizations of the control scheme, we will see below that we can indeed stabilize β in a range of values between the two universal limits, 1/4 and 1/3, over at least one to two decades in time.

This letter is organized as follows. We first introduce the KPZ equation and our numerical solution scheme, accompanied by a representative data set for the growing surface roughness, and specifically its growth exponent, in the absence of control. Next, we implement two types of time-delayed feedback control and demonstrate how reasonably accurate values of the (effective) growth exponent can be achieved. We conclude with a summary and a discussion of open questions.

The KPZ equation. If overhangs and bulk fluctuations can be neglected, growth phenomena are often modelled in terms of nonlinear stochastic partial differential equations. A single-valued variable, $h(x,t)$, denotes the height of the surface above a reference plane and fluctuates as a function of time t and position x (measured in this d-dimensional plane). The simplest such equation is the KPZ equation [17] which describes the growth of a surface in the absence of any conservation laws:

$$\partial_t h(x,t) = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}(\nabla h)^2 + \eta(x,t) \qquad (1)$$

Here, $\nu > 0$ denotes an interface smoothing term, associated with a surface tension; the nonlinear coupling λ reflects the strength of lateral growth, and $\eta(x,t)$ models the height fluctuations due to random deposition of material. An overall, constant growth velocity has already been eliminated, by transforming into a suitable comoving frame. Focusing on large-scale, long-time properties of the surface, it is sufficient to consider Gaussian white noise, i.e.,

$$\begin{aligned} < \eta(x,t) > &= 0 \\ < \eta(x,t)\eta(x',t') > &= 2D\delta^d(x-x')\delta(t-t') \qquad (2) \end{aligned}$$

The KPZ equation has been discussed in many different contexts, including thin film growth [18, 19, 20, 21], fluctuating hydrodynamics [22], driven diffusive systems

*Electronic address: block@itp.physik.tu-berlin.de; Fax: +49-(0)30-314-21130

[23, 24], tumor growth in biophysics [25, 26], propagating fire fronts [27, 28] and econophysics [29]. In the following, we will use the language of surface growth, but our findings are easily translated into these other contexts and just as relevant there.

For simplicity, we restrict ourselves to one spatial dimension. We monitor the time-dependence of the (root mean square) surface roughness w, defined by

$$w^2(L,t) = \frac{1}{L}\left\langle \sum_x [h(x,t) - \bar{h}(t)]^2 \right\rangle \tag{3}$$

Here, L denotes the system size, and $\bar{h}(t) \equiv L^{-1}\sum_x h(x,t)$ is the mean surface height at time t. Configurational averages are denoted by $\langle ... \rangle$. The sum over x anticipates the space discretization associated with the numerical integration scheme. It is well known that w obeys scaling in the form $w(L,t) = L^\alpha f(t/L^z)$ where f is a scaling function and α and z denote the roughness and dynamic exponents, respectively [30]. In the saturation regime $t/L^z \gg 1$, f approaches a constant so that $w \sim L^\alpha$ becomes independent of time. In contrast, in the growth regime $t/L^z \ll 1$, the width grows as a power of time, $w \sim t^\beta$, with a growth exponent β. Consistency with the general scaling form imposes an exponent identity, $\beta = \alpha/z$. The values for the exponents α, β, and z are universal. Two universality classes can be distinguished. If the nonlinear term of the KPZ equation vanishes ($\lambda = 0$), the equation reduces to the exactly soluble Edwards-Wilkinson (EW) equation [31], with $\alpha = 1/2$ and $z = 2$ whence $\beta = 1/4$. In contrast, any *nonzero* value of λ belongs to the KPZ universality class with $\alpha = 1/2$ and $z = 3/2$ whence $\beta = 1/3$ [17]. These values give us some benchmarks against which we can check our numerical scheme. We use a forward-backward Euler method [32] to solve the stochastic differential equations numerically. The associated discretization captures certain prefactors much more accurately than the standard (naive) scheme [33]. Our integration parameters are $\nu = 0.1$ and $D = 0.5$; the space discretization is set to $\Delta x = 1$, with $L = 1024$ and 4096, and the time increment is set at $\Delta t = 10^{-3}$. Finally, λ varies between 0.00 and 0.25. The upper cutoff is chosen so as to avoid numerical instabilities.

Figure 1 shows a scaling plot for $w(L,t)$ before any control schemes are implemented. Data for three different values of λ are shown. The roughness exponent α is consistent with 1/2, independent of λ, as expected. For $\lambda = 0$, we see excellent data collapse with the EW scaling exponents, and the KPZ exponents are confirmed for the largest λ. The latter should be universal, for all $\lambda \neq 0$; however, for $0 < \lambda < 0.25$, strong crossover effects between EW and KPZ behavior are observed. Remarkably, this crossover manifests itself as a surprisingly clean power law, with a λ-dependent *effective* growth exponent β *below* 1/3. Eventually, the asymptotic value (1/3) is reached, but only after an L- and λ-dependent crossover time.

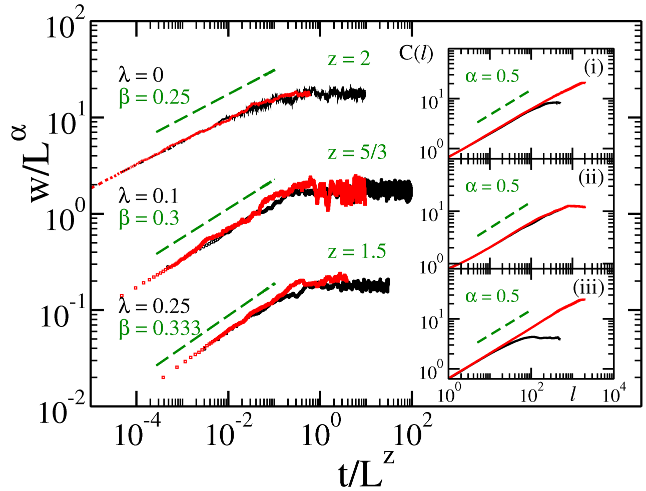

Figure 1: (color online). Scaling plots of the roughness $w(L,t)$ and height-height correlation function $C(l) \equiv \langle h(l,t)h(0,t)\rangle \sim l^\alpha$ (insets) for system sizes $L = 1024$ (red) and $L = 4096$ (black) and three choices of λ: (i) $\lambda = 0$ (shifted by a factor 100), (ii) $\lambda = 0.1$ (shifted by a factor 10), (iii) $\lambda = 0.25$. $\nu = 0.1$ for all data sets. The green broken lines provide guides to the eye.

Systems with control. We now turn to possible control mechanisms. For chemical vapor deposition of silica films, there is some experimental evidence [18] that the lateral growth velocity is related to the temperature, via a temperature-depending sticking probability. In other words, λ – and hence effective growth exponents – can be controlled via the temperature. For our differential equation, we tune λ directly, in order to stabilize a *desired* effective growth exponent, β_0. In detail, the scheme is as follows. First, we choose the desired value of the growth exponent, β_0, and select an appropriate time delay τ. Generating sufficiently many samples of $h(x,t)$, we record $w(t-\tau)$ and $w(t)$ (the argument L will be omitted from now on). The *local* exponent β_{local} at time t is defined as

$$\beta_{local}(t) \equiv \frac{\log w(t) - \log w(t-\tau)}{\log t - \log(t-\tau)} \tag{4}$$

Depending on the sign and value of $\beta_{local}(t) - \beta_0$, we adjust the nonlinear coupling, λ, of the KPZ equation, as follows. First, we introduce a control function $F(t)$. For *digital* control, we define

$$F(t) \equiv \begin{cases} a, & if\ \beta_{local} \leq \beta_0 \\ -a, & if\ \beta_{local} > \beta_0 \end{cases} \tag{5}$$

where the parameter a defines the control "bit", i.e., the amount by which λ changes at each control step. Alternatively, we also investigate a *differential* method for

which

$$F(t) \equiv K(\beta_0 - \beta_{local}) \tag{6}$$

and K sets the amplitude of the control strength. Given one of the two choices of $F(t)$, the control scheme sets in at time t_0. From then on, the nonlinearity λ is updated at times $t_n \equiv t_0 + n\tau$, $n = 1, 2, ...$, starting from an initial value λ_0, according to

$$\lambda(t) = \begin{cases} \lambda_0, & if\ t < t_0 \\ \lambda(t-\tau) + F(t), & if\ t = t_n \\ \lambda(t_n), & if\ t_n < t < t_{n+1} \end{cases} \tag{7}$$

Our scheme is successful if $\beta_{local}(t)$ approaches β_0 and then settles at the desired value within a reasonable time frame after the control has been activated.

Some comments are in order. Starting from a random initial condition, we first choose a starting value, λ_0, for the nonlinearity and integrate the KPZ equation without control up to time t_0, in order to eliminate transients. A reasonably stable growth regime is achieved around $t_0 \sim 10$, independent of L (provided L is not too small, i.e., $L \geq 64$). Then, we turn on the control, following either the digital or the differential scenario. Regarding the choice of the time delay τ, it must be large enough compared to the time increment Δt so as not to interfere with the integration procedure, but small enough to provide responsive control. We find that we get good results for a time delay $0.1 < \tau < 1.0$. Similarly, we choose the control amplitudes a and K such that the increments in λ are small compared to λ_0 but large enough to generate a noticeable response. For example for $\tau = 1.0$, choosing a in the range $[0.002, 0.02]$ and K in the range $[0.005, 0.05]$ provides the best results.

Results. Figures 2 and 3 show our results. Starting from three initial values of λ_0, namely, 0, 0.1, and 0.25, we attempt to stabilize the effective growth exponent at $\beta_0 = 0.29$, mid-way between the KPZ and EW values. Irrespective of λ_0, we find that both digital and differential control result in an effective growth exponent very close to the desired value, over at least a decade of integration time ($100 \lesssim t \lesssim 1000$).

For sufficiently large time, the control function $\lambda(t)$ appears to approach a constant value, close to 0.15. However, the details of the approach depend on the initial λ_0. For strong initial nonlinearity $\lambda_0 = 0.25$, $\lambda(t)$ is approximately monotonically decreasing, apart from significant fluctuations. For both weak and vanishing initial coupling, $\lambda(t)$ approaches its "limit" from below. This behavior is observed for both, digital and differential, control. We tested several other choices of $0.25 \leq \beta_0 < 0.33$ and λ_0, and found similar behavior.

Experimentally, it is usually desirable to achieve small roughness. To push our control schemes to the limit, we test the most extreme case, namely, $\beta_0 = 0.25$ with large initial nonlinearity $\lambda_0 = 0.25$. Before the control sets in, the roughness grows considerably faster than $t^{0.25}$. As soon as the control sets in, $\lambda(t)$ decreases quite dramatically, leading to a reduction of the effective β. However, over the time period considered ($t \leq 1000$), it never decreases far enough to reach the desired 0.25.

Finally, we note that it is not possible to achieve exponent values *outside* the interval $[0.25, 0.33]$. Choosing $\beta_0 > 0.33$ generates unbounded growth of the control function $\lambda(t)$, accompanied by instabilities in the integration routine. Similarly, $\beta_0 < 0.25$ quickly leads to large *negative* values of $\lambda(t)$ which tend to favor KPZ exponents (since the sign of λ plays no role). As a result, $\lambda(t)$

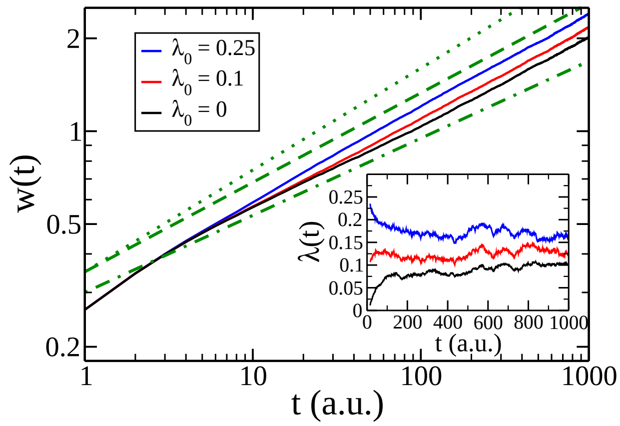

Figure 2: (color online). Roughness and control function (inset) evolution for digital time delayed feedback control for strong (blue), weak (red), and zero (black) initial nonlinearity λ_0. The desired effective growth exponent is set at $\beta_0 = 0.29$. To provide a comparison, the straight (green) lines have slopes 0.33 (dotted), 0.29 (dashed), and 0.25 (dash-dotted). All data sets are obtained with $\nu = 0.1$ and $a = 0.01$.

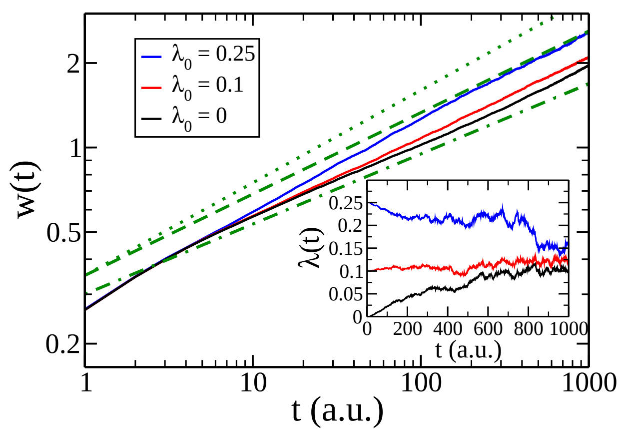

Figure 3: (color online). Roughness and control function (inset) evolution for differential time delayed feedback control for strong (blue), weak (red), and zero (black) initial nonlinearity λ_0. The desired effective growth exponent is set at $\beta_0 = 0.29$. To provide a comparison, the straight (green) lines have slopes 0.33 (dotted), 0.29 (dashed), and 0.25 (dash-dotted). All data sets are obtained with $\nu = 0.1$ and $K = 0.02$.

becomes even more negative until a numerical instability occurs. To avoid this instability, we also implemented a symmetrized version of control (with $a \to -a$ when $\lambda(t) < 0$). In this case, $\lambda(t)$ approaches zero and fluctuates about it, so that the effective exponent settles at 0.25.

Conclusions. To summarize, both digital and differential control are rather successful at stabilizing effective growth exponents in the KPZ equation. For the relatively small system sizes used here, these exponents can be tuned in the range $[0.25, 0.33]$, i.e., within the limits set by the EW and the KPZ equation, respectively. Let us emphasize again that only the values $1/4$ (for $\lambda = 0$) and $1/3$ (for any $\lambda \neq 0$) correspond to true asymptotic exponents; for larger system sizes and longer integration times, these emerge clearly. However, for small systems, we observe surprisingly clean effective growth exponents which appear to depend monotonically on the magnitude of the nonlinearity. Hence, it is possible to choose a desired reference exponent β_0 and implement a time-delayed control of the nonlinearity in such a way that the effective exponent first approaches β_0 and then stabilizes at that value for a significant length of time (roughly, $10^2 \lesssim t \lesssim 10^3$, in our units). In all simulations, the (stationary) roughness remains constant at $\alpha = 0.5$, reflecting the value $1/2$ which is common to both the EW and the KPZ equation. The control protocol itself is independent from the dimension of the surface. Work is in progress to test other growth equations and to extend the KPZ study to 2+1 dimensions. We thus hope to develop a control tool which might also be useful in experimental setups. The work of Ojeda et al [18, 19] gives some indications that the KPZ nonlinearity is tunable via the temperature. Hence, it seems feasable to implement a time-delayed feedback loop and stabilize desired growth exponents by suitable adjustments of the temperature.

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